

eSpyMath: Prealgebra**Table of Contents**

Preface	1
About eSpyMath Prealgebra Workbook	2
How to use this book	3
Chapter 1. Number Sense and Operations	6
1.1. Key Topics on Number Sense and Operations	7
1.2. Key Questions.....	28
1.3. Solutions.....	36
Chapter 2. Algebraic Expressions and Equations	39
2.1. Key Topics on Algebraic Expressions and Equations	40
2.2. Key Questions.....	53
2.3. Solutions.....	59
Chapter 3. Solving Equations and Inequalities.....	62
3.1. Key Topics on Solving Equations and Inequalities.....	63
3.2. Key Questions.....	79
3.3. Solutions.....	89
Chapter 4. Ratios, Proportions, and Percentages	92
4.1. Key Topics on Ratios, Proportions, and Percentages.....	93
4.2. Key Questions.....	109
4.3. Solutions.....	119
Chapter 5. Linear Functions and Graphs.....	123
5.1. Key Topics on Linear Functions and Graphs	124
5.2. Key Questions.....	152
5.3. Key Solutions	160
Chapter 6. Systems of Equations and Inequalities.....	164
6.1. Key Topics on Systems of Equations and Inequalities	165
6.2. Key Questions.....	188
6.3. Solutions.....	195
Chapter 7. Geometric Concepts and Shapes	198
7.1. Key Topics on Geometric Concepts and Shapes.....	199
7.2. Key Questions.....	223
7.3. Solutions.....	230

Chapter 8. Probability and Data Analysis.....	234
8.1. Key Topics on Probability and Data Analysis	235
8.2. Key Questions.....	252
8.3. Solutions.....	261
Appendix. Prealgebra Foundation Concepts.....	265
End of the Workbook	279

Chapter 1. Number Sense and Operations

- 1-1. **Whole Numbers, Integers, Rational Numbers, and Irrational Numbers**
- 1-2. **Properties of Operations (Commutative, Associative, Distributive)**
- 1-3. **Adding, Subtracting, Multiplying, and Dividing Real Numbers**
- 1-4. **Absolute Value and Its Properties**
- 1-5. **Exponents, Including Zero and Negative Exponents**
- 1-6. **Square Roots and Cube Roots**
- 1-7. **Scientific Notation**
- 1-8. **Order of Operations (PEMDAS/BODMAS)**
- 1-9. **Converting Between Fractions, Decimals, and Percents**
- 1-10. **Comparing & Ordering Number Forms**
- 1-11. **Prime and Composite Numbers**
- 1-12. **Factors and Multiples (Divisibility Tests)**
- 1-13. **Prime Factorization**
- 1-14. **Least Common Multiple (LCM) and Greatest Common Divisor (GCD)**
- 1-15. **Perfect Squares and Perfect Cubes**
- 1-16. **Rounding Numbers and Significant Figures**
- 1-17. **Evaluating Expressions**
- 1-18. **The Real Number System**
- 1-19. **Comparing & Ordering Number Forms (Review from previous list)**
- 1-20. **Fundamental properties of equality and operations in mathematics**

1.1. Key Topics on Number Sense and Operations

1-1. Whole Numbers, Integers, Rational and Irrational Numbers

- **Natural Numbers:** Natural numbers are the numbers you use for counting, such as 1, 2, 3, 4, and so on. They are the most basic set of numbers.
- **Whole Numbers:** Whole numbers include all the natural numbers along with zero. So, the set of whole numbers is 0, 1, 2, 3, 4, and so on. These numbers are used in the base ten number system.
- **Integers:** Integers are positive and negative whole numbers, including zero.
- **Rational Numbers:** Numbers that can be expressed as fractions.
- **Irrational Numbers:** Numbers that cannot be expressed as fractions. They have non-repeating, non-terminating decimal representations.

Ex 1. Is -3 an integer or a rational number?	Ex 2. Is $\sqrt{2}$ a rational number?
Ex 3. Read 15,622,396	Ex 4. Is 0 a whole number?
Ex 5. Is π an irrational number?	

Solution

1. 3 is **both an integer and a rational number**, as it can be expressed as a fraction of two integers (-3/1).
2. No, $\sqrt{2}$ cannot be expressed as a fraction of two integers, making it an **irrational number**.
3. “Fifteen million six hundred twenty-two thousand three hundred ninety-six”

Millions			Thousands			Hundreds		
<i>hundreds</i>	<i>tens</i>	<i>units</i>	<i>hundreds</i>	<i>tens</i>	<i>units</i>	<i>hundreds</i>	<i>tens</i>	<i>units</i>
1	5	6	2	2	3	9	9	6

4. Yes, 0 is a whole number.
5. Yes, π an irrational number.

1-2. Properties of Operations (Commutative, Associative, Distributive)

- **Commutative Property:** $a+b=b+a$ and $ab=ba$
- **Associative Property:** $(a+b)+c=a+(b+c)$ and $(ab)c=a(bc)$
- **Distributive Property:** $a(b+c)=ab+ac$

Ex 1. Use the associative property to simplify $2+(3+4)$	Ex 2. Simplify $3(4+5)-2$ using distributive property
Ex 3. Use the commutative property to rewrite $5+9$	Ex 4. Use the distributive property to simplify $3(x+4)$
Ex 5. Use the distributive property to simplify $(x+4)5$	

Solution

1. $2+(3+4)=(2+3)+4=9$.
2. $3(4+5)-2=3\times 4+3\times 5-2=12+15-2=25$.
3. $9+5$.
4. $3(x+4)=3x+12$.
5. $(x+4)5=5x+20$

1-3. Adding, Subtracting, Multiplying, and Dividing Real Numbers

Ex 1. Calculate $3 - 2(5 + 3)$	Ex 2. Calculate $(-3/4) \times (8/5)$
Ex 3. Calculate $3.5 + (-2.5)$	Ex 4. Calculate $(-3/4) \div (8/5)$
Ex 5. Calculate $3 - 2 \times 8 \div 4$	Ex 6. Calculate $3 - 8 \times 4 \div 2$
Ex 7. Calculate $3 - 8 \times (4 \div 2)$	

Solution

1. $3 - 2(5 + 3) = 3 - 2 \times 8 = -13$
2. $(-3/4) \times (8/5) = \frac{-3}{4} \times \frac{8}{5} = -\frac{24}{20} = -\frac{6}{5}$
3. $3.5 + (-2.5) = 1$
4. $(-3/4) \div (8/5) = \frac{-3}{4} \times \frac{5}{8} = -\frac{15}{32}$
5. $3 - 2 \times 8 \div 4 = 3 - 16 \div 4 = 3 - 4 = -1$
6. $3 - 8 \times 4 \div 2 = 3 - 32 \div 2 = 3 - 16 = -13$
7. $3 - 8 \times (4 \div 2) = 3 - 8 \times 2 = -13$

1-4. Absolute Value and Its Properties

- **Formula:** $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$
- **Note:** The absolute value of a number is its distance from zero on the number line, regardless of direction.

Ex 1. Find the absolute value of -7 .	Ex 2. Simplify $ -3 + 4 - 7 $
Ex 3. Find a value when $ a = 3$	Ex 4. Find a value when $ -a = 3$
Ex 5. Find a value when $ 2x - 1 = 3$	Ex 6. Find a value when $ 1 - 2x = 3$

Solution

1. $|-7| = 7$
2. $|-3| + |4 - 7| = 3 + |-3| = 3 + 3 = 6$
3. $a = 3$ or -3
4. $a = 3$ or -3
5. $2x - 1 = 3$ or $2x - 1 = -3 \Rightarrow x = 2$ or $x = 1$
6. $1 - 2x = 3$ or $1 - 2x = -3 \Rightarrow x = 1$ or $x = 2$

1-5. Exponents, Including Zero and Negative Exponents

- **Formula for Positive Exponents:** $a^n = a \times a \times \dots \times a$ (n times)
- **Zero Exponent:** $a^0 = 1$ (for any non-zero 'a')
- **Negative Exponents:** $a^{-n} = \frac{1}{a^n}$
- **Note:** The base 'a' is multiplied by itself 'n' times. For negative exponents, the base is in the denominator.

Ex 1. Calculate 2^3	Ex 2. Calculate 2^{-3}
Ex 3. Calculate $\left(\frac{1}{2}\right)^{-1}$	Ex 4. Calculate $\left(\frac{1}{2}\right)^3$
Ex 5. Calculate $\left(\frac{1}{2}\right)^{-3}$	

Solution

1. $2^3 = 2 \times 2 \times 2 = 8$.
2. $2^{-3} = 1/(2^3) = 1/8$.
3. $\left(\frac{1}{2}\right)^{-1} = 2$
4. $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$
5. $\left(\frac{1}{2}\right)^{-3} = (2^{-1})^{-3} = 2^3 = 8$

1-6. Square Roots and Cube Roots

- **Square Root:** \sqrt{x} is a number that, when multiplied by itself, gives 'x'.
- **Cube Root:** $\sqrt[3]{x}$ is a number that, when used in a product three times, gives 'x'.
- **Note:** Square roots and cube roots can be both positive and negative since both $a^2 = a \times a$ and $(-a)^2 = (-a) \times (-a)$ result in the same number.

Ex 1. Find $\sqrt{9}$	Ex 2. Find $\sqrt[3]{27}$
Ex 3. Find $\sqrt{(-2)^2}$	Ex 4. Find $\sqrt[3]{-27}$
Ex 5. Find $\sqrt{(-2)^2}$	

Solution

1. $\sqrt{9} = 3$.
2. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$.
3. $\sqrt{(-2)^2} = 2$
4. $\sqrt[3]{-27} = -3$
5. $\sqrt{(-2)^2} = 2$

1-7. Scientific Notation

- **Formula:** $a \times 10^n$, where $1 \leq |a| < 10$ and 'n' is an integer.
- **Note:** Used to express very large or very small numbers in a compact form. 'a' is the significand or mantissa, and 'n' is the order of magnitude.

Ex 1. Write 3000 in scientific notation.	Ex 2. Multiply 2×10^4 by 3×10^3 in scientific notation.
Ex 3. Write 3500 in scientific notation.	Ex 4. Multiply 20000 by 6000 in scientific notation.
Ex 5. Add 20000 by 6000 in scientific notation.	Ex 6. Write 0.00035 in scientific notation.

Solution

1. $3000 = 3 \times 10^3$
2. $(2 \times 10^4) \times (3 \times 10^3) = 6 \times 10^{4+3} = 6 \times 10^7$
3. $3500 = 3.5 \times 10^3$
4. $(2 \times 10^4) \times (6 \times 10^3) = 12 \times 10^{4+3} = 1.2 \times 10^8$
5. $(2 \times 10^4) + (6 \times 10^3) = (20 \times 10^3) + (6 \times 10^3) = (20 + 6) \times 10^3 = 26 \times 10^3 = 2.6 \times 10^4$
6. $0.00035 = 3.5 \times 10^{-4}$

1-8. Order of Operations (PEMDAS/BODMAS)

- **Formula:** Parentheses/Brackets, Exponents/Orders (i.e., powers and roots), Multiplication and Division (from left to right), Addition and Subtraction (from left to right).
- **Note:** Operations inside parentheses/brackets are performed first, followed by exponents, then multiplication and division, and finally, addition and subtraction.

Ex 1. Simplify $2 + 3 \times 4$	Ex 2. Simplify $4 + 16 \div (2^2) \times 3$
Ex 3. Simplify $7 - 32 \div 8 \times 4 + 12$	Ex 4. Simplify $7 - 32 \div (8 \times 4) + 12$
Ex 5. Simplify $18 \div 3 - 7 + 2 \times 3$	Ex 6. Simplify $18 - 6 \div 3 + 2 \times 3$
Ex 7. Simplify $-2[(4 - 2 \times 7) \div 5] - 2 \times 24 \div 6$	

Solution

1. $2 + (3 \times 4) = 2 + 12 = 14$
2. $4 + 16 \div (2^2) \times 3 = 4 + 16 \div 4 \times 3 = 4 + 4 \times 3 = 4 + 12 = 16$
3. $7 - (32 \div 8) \times 4 + 12 = 7 - (4 \times 4) + 12 = 7 - 4 = 3$
4. $7 - 32 \div (8 \times 4) + 12 = 7 - 1 + 12 = 18$
5. $(18 \div 3) - 7 + (2 \times 3) = 6 - 7 + 6 = 5$
6. $18 - (6 \div 3) + (2 \times 3) = 18 - 2 + 6 = 22$
7. $-2[(4 - 2 \times 7) \div 5] - 2 \times 24 \div 6 = -2[(-10) \div 5] - (2 \times 24 \div 6) = -2 \times (-2) - (48 \div 6) = 4 - 8 = -4$

1-9. Converting Between Fractions, Decimals, and Percents

- **To Convert a Fraction to a Decimal:** Divide the numerator by the denominator.
- **To Convert a Decimal to a Percent:** Multiply the decimal by 100 and add the percent symbol (%).
- **To Convert a Percent to a Decimal:** Divide the percent value by 100.
- **Note:** Understanding these conversions is crucial for solving problems involving parts of a whole in different representations.

Ex 1. Convert 0.75 to a percent.	Ex 2. Convert $\frac{3}{8}$ to a decimal.
Ex 3. Convert the fraction $\frac{2}{5}$ to a decimal.	Ex 4. What is the percent value of the decimal 0.5?
Ex 5. Express the fraction $\frac{3}{8}$ as a percent.	Ex 6. Convert the percent 150% to a decimal.
Ex 7. Convert 1.25 to a percent.	

Solution

1. $0.75 \times 100\% = 75\%$
2. $3 \div 8 = 0.375$
3. $2 \div 5 = 0.4$
4. $0.5 \times 100 = 50\%$
5. $3 \div 8 = 0.375; 0.375 \times 100 = 37.5\%$
6. $150\% \div 100 = 1.5$
7. $1.25 \times 100\% = 125\%$

1-10. Comparing & Ordering Number Forms

Ex 1. Order the numbers $0.3, \frac{1}{4}, 25\%$	Ex 2. Compare $\sqrt{2}$ and 1.4
Ex 3. Compare and order the following numbers: $1/4, 0.5, 25\%$	Ex 4. Arrange the numbers in ascending order: $3/8, 0.6, 75\%$
Ex 5. Compare the values and arrange the numbers in ascending order: $2/3, 0.67, 70\%$	

Solution

1. First, convert all to decimals: $0.3, 0.25, 0.25$. Thus, $0.25 = \frac{1}{4} = 25\% < 0.3$. Answer: $\frac{1}{4}, 25\%, 0.3$
2. Since $\sqrt{2} \approx 1.414$, $\sqrt{2} > 1.4$.
3. Solution
 - Convert $1/4$ to a decimal: $1 \div 4 = 0.25$
 - Convert 0.5 to a percent: $0.5 \times 100 = 50\%$
 - Compare: $0.25 < 0.5 < 25\%$
 - $\frac{1}{4}, 25\%, 0.5$
4. Solution
 - Convert $3/8$ to a decimal: $3 \div 8 = 0.375$
 - Convert 75% to a decimal: $75\% \div 100 = 0.75$
 - Compare: $0.375 < 0.6 < 0.75$
 - $\frac{3}{8}, 0.6, 75\%$
5. Solution
 - Convert $2/3$ to a decimal: $2 \div 3 \approx 0.6667$
 - Convert 70% to a decimal: $70\% \div 100 = 0.7$
 - Compare: $2/3 = 0.67 = 0.7$
 - Order: $2/3, 0.67, 70\%$

1-11. Prime and Composite Numbers

Ex 1. Is 7 a prime or a composite number?	Ex 2. Is 51 a prime or a composite number?
Ex 3. Is 1 a prime or a composite number?	Ex 4. Is 97 a prime or a composite number?

Solution

1. 7 has only two distinct positive divisors: 1 and itself. Therefore, 7 is a **prime number**.
2. 51 can be divided by 1, 3, 17, and 51. Since it has more than two divisors, 51 is a **composite number**.
3. Neither.
4. 97 is a **prime number** because it has only two distinct positive divisors: 1 and 97.

1-12. Factors and Multiples (Divisibility Tests)

- A whole number is **divisible by 2** if its last digit is 0, 2, 4, 6, or 8.
- A whole number is **divisible by 3** if the sum of its digits is divisible by 3.
- A whole number is **divisible by 4** if its last two digits are divisible by 4.
- A whole number is **divisible by 5** if its last digit is 0 or 5.
- A whole number is **divisible by 6** if it is divisible by both 2 and 3.
- A whole number is **divisible by 8** if its last three digits are divisible by 8.
- A whole number is **divisible by 9** if the sum of its digits is divisible by 9.
- A whole number is **divisible by 10** if its last digit is 0.

Ex 1. Find all factors of 12.	Ex 2. What is the least common multiple (LCM) of 12 and 18?
Ex 3. Does 1,248 have 8 as a factor?	Ex 4. Does 1,248 have 6 as a factor?
Ex 5. Does 111,111,111 have 9 as a factor?	

Solution

1. The factors of 12 are 1, 2, 3, 4, 6, and 12.
2. The prime factorization of 12 is $2^2 \times 3$, and of 18 is 2×3^2 .
 - The LCM is $2^2 \times 3^2 = 36$.
3. Yes, look at the last three digits, 248, which is divisible by 8, so 1,248 is divisible by 8.
4. Yes, 1248 is an **even number** since it ends in 4, and the sum of its digits, $1+2+4+8=15$, is a **multiple of 3**. Thus, confirming that the number is also a **multiple of 6**.
5. Yes, the sum of the digits of 111,111,111 is: $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 9$. So, the sum of the digits (9) is divisible by 9.

1-13. Prime Factorization

- **Formula:** Not applicable as a specific **Formula**, but the process involves dividing the number by prime numbers starting from the smallest (2, 3, 5, 7, 11, ...) until the quotient is 1.
- **Note:** Prime factorization is unique for every number, excluding the order of the factors (Fundamental Theorem of Arithmetic).

Ex 1. Find the prime factorization of 30.	Ex 2. Find the prime factorization of 84.
Ex 3. Find the prime factorization of 343.	Ex 4. What is the prime factorization of 45?
Ex 5. Determine the prime factorization of 100.	

Solution

1. $30 = 2 \times 3 \times 5$
2. $84 = 2^2 \times 3 \times 7$
3. $343 = 7^3$
4. The prime factorization of 45 is $3^2 \times 5$
5. The prime factorization of 100 is $2^2 \times 5^2$

1-14. Least Common Multiple (LCM) and Greatest Common Divisor (GCD)

- **GCD (a, b):** The largest number that divides both 'a' and 'b' without leaving a remainder.
- **LCM (a, b):** The smallest non-zero common multiple of 'a' and 'b'.
- **Note:** For two numbers, the product of the GCD and LCM is equal to the product of those numbers: $a \times b = GCD(a, b) \times LCM(a, b)$.

Ex 1. Find the GCD of 8 and 12.	Ex 2. What is the GCD of 8 and 12?
Ex 3. Find the LCM of 4 and 5.	Ex 4. Find the LCM of 20 and 30.
Ex 5. Calculate the LCM and GCD of 6 and 9.	Ex 6. Calculate the LCM and GCD of $6x^2y$ and $9xy^2$.

Solution

1. The factors of 8 are 1, 2, 4, 8. The factors of 12 are 1, 2, 3, 4, 6, 12. The GCD is 4.
2. The GCD of 8 and 12 is 4.
3. The LCM of 4 and 5 is 20.
4. The prime factorization of 20 is $2^2 \times 5$, and of 30 is $2 \times 3 \times 5$. The LCM is $2^2 \times 3 \times 5 = 60$.
5. The LCM of 6 and 9 is 18, and the GCD is 3.
6. The LCM of 6 and 9 is $18x^2y^2$, and the GCD is $3xy$.

1-15. Perfect Squares and Perfect Cubes

Ex 1. Is 16 a perfect square?	Ex 2. Is 64 a perfect cube?
Ex 3. Which number is both a perfect square and a perfect cube, except 1?	Ex 4. Between which two consecutive numbers does $\sqrt{50}$ lie?
Ex 5. What is the cube of 3?	Ex 6. Identify a number between 1 and 10 that is both a perfect square and a perfect cube.

Solution

1. Yes, because $4^2 = 16$.
2. Yes, because $4^3 = 64$.
3. 64 (since $8^2 = 64$ and $4^3 = 64$)
4. Between 7 and 8 (since $7^2 = 49$ and $8^2 = 64$)
5. The cube of 3 is $3^3 = 27$
6. The number 1 is both a perfect square (1^2) and a perfect cube (1^3).

1-16. Rounding Numbers and Significant Figures

- Rounding numbers is a way of making numbers simpler and easier to work with by approximating them to a certain degree of accuracy.
- The leading zeros to the left of the first non-zero digit (in this case, there are none) are not **significant**.
- All non-zero digits are **significant**, which means 2, 3, and 4 are significant.
- Any zeros between significant digits are **significant**, which includes the zero between 3 and 4.
- Trailing zeros to the right of the decimal point are **significant**, which means the final zero is significant.

Ex 1. Round 3.14159 to two decimal places.	Ex 2. Round 123456 to 3 significant figures.
Ex 3. Round 123.456 to the nearest whole number.	Ex 4. How many significant figures are in the number 0.002030?
Ex 5. How many significant figures are in the number 2.0304?	Ex 6. How many significant figures are in the number 2.0340?

Solution

1. 3.14
2. 123000
3. 123
4. There are 4 significant figures.
5. There are 5 significant figures.
6. There are 5 significant figures.

1-17. Evaluating Expressions

Ex 1. Evaluate $2x + 3$ when $x = 3$	Ex 2. Evaluate $3x^2 - 2x + 1$ when $x = -2$
Ex 3. Evaluate the expression $2x + 3$ when $x = -5$	Ex 4. What is the value of $3y - 4$ when $y = 4$?
Ex 5. Evaluate $x^2 + 2x - 1$ for $x = 3$	

Solution

1. $2(3) + 3 = 9$.
2. $3(-2)^2 - 2(-2) + 1 = 12 + 4 + 1 = 17$.
3. $2(-5) + 3 = -10 + 3 = -7$
4. $3(4) - 4 = 8$
5. $3^2 + 2(3) - 1 = 14$

1-18. The Real Number System

Ex 1. Classify the number 0	Ex 2. Classify the number π
Ex 3. Classify the number $\sqrt{3}$	Ex 4. Classify the number $\sqrt{3} + 2$
Ex 5. Classify the number $\sqrt{9}$	Ex 6. Classify the number $\frac{1}{3}$
Ex 7. Classify the number $\frac{1}{\sqrt{3}}$	

Solution

1. 0 is a whole number, an integer, a rational number, and a real number.
2. π is an irrational number.
3. $\sqrt{3}$ is an irrational number.
4. $\sqrt{3} + 2$ is an irrational number.
5. $\sqrt{9} = 3$ is a natural number, a whole number, an integer, a rational number
6. $\frac{1}{3}$ is a rational number.
7. $\frac{1}{\sqrt{3}}$ is an irrational number.

1-19. Comparing & Ordering Number Forms (Review from previous list)

Ex 1. Which is larger: 0.5 or $\frac{1}{3}$?	Ex 2. Order the numbers $\sqrt{2}$, 1.5, and $\frac{3}{2}$
Ex 3. If the set is ordered from least to greatest, which value fits in the box? $\left\{3^{-2}, x, \frac{1}{4}\right\}$	Ex 4. Arrange in ascending order: $1/3$, $3/10$, 33% (smallest to largest)
Ex 5. Order the numbers: $3/10$, 0.3, 30%	

Solution

1. 0.5 is larger than $\frac{1}{3}$ (or 0.333...).
2. Since $\sqrt{2} \approx 1.414$, and $1.5 = \frac{3}{2}$, the order is $\sqrt{2} < 1.5 = \frac{3}{2}$.
3. $3^{-2} = \frac{1}{9}$, so x could be any value between $\frac{1}{9}$ and $\frac{1}{4}$, like $\frac{1}{8}$.
4. Solution: $3/10 < 33\% < 1/3$
 - Convert $1/3$ to a decimal: $1 \div 3 = 0.3333$ (rounded to 0.33)
 - Convert 33% to a decimal: $33 \div 100 = 0.33$
 - Compare: $3/10 = 0.3$
5. Solution: $3/10 = 0.3 = 30\%$
 - Convert $3/10$ to a decimal: $3 \div 10 = 0.3$
 - Convert 30% to a decimal: $30\% \div 100 = 0.3$
 - Compare: $0.3 = 0.3$

1-20. Fundamental properties of equality and operations in mathematics

- **Reflexive Property:** For any real number a , $a=a$
- **Symmetric Property:** If $a=b$, then $b=a$
- **Transitive Property:** If $a=b$ and $b=c$, then $a=c$
- **Substitution Property:** If $a=b$, then a can be replaced by b
- **Commutative Property:** For addition, $a+b=b+a$, and for multiplication, $ab=ba$
- **Associative Property:** For addition, $(a+b)+c=a+(b+c)$, and for multiplication, $(ab)c=a(bc)$
- **Distributive Property:** $a(b+c)=ab+ac$ and $(b+c)a=ba+ca=ab+ac$
- **Multiplicative Property of Zero:** $a \times 0 = 0$ and $0 \times a = 0$
- **Additive Identity Property:** $a+0=a$ and $0+a=a$
- **Multiplicative Identity Property:** $a \times 1=a$ and $1 \times a=a$
- **Addition Property of Equality:** If $a=b$, then $a+c=b+c$
- **Subtraction Property of Equality:** If $a=b$, then $a-c=b-c$
- **Multiplication Property of Equality:** If $a=b$, then $ac=bc$
- **Division Property of Equality:** If $a=b$ and $c \neq 0$, then $\frac{a}{c}=\frac{b}{c}$
- **Additive Inverse (Axiom of Opposites):** $a+(-a)=0$
- **Multiplicative Inverse (Axiom of Reciprocals):** If $a \neq 0$, then $a \times \frac{1}{a}=1$
- **Property of the Opposite of a Sum:** $-(a+b)=(-a)+(-b)$
- **Property of the Reciprocal of a Product:** If $a \neq 0$ and $b \neq 0$, then $\frac{1}{ab}=\frac{1}{a} \times \frac{1}{b}$
- **Property of Proportions:** If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$ (The cross products are equal)

Ex 1. Is it true that for any real number x , $x=x$? Please specify the purpose for which the property is being utilized.

Ex 2. If $a=4$, what is the value of b if $b=a$? Please specify the purpose for which the property is being utilized.

Ex 3. If $p=2$, $q=p$, and $r=q$, what is the value of r in relation to p ? Please specify the purpose for which the property is being utilized.

Ex 4. If $m=5$, can we replace m with 5 in any mathematical expression involving m ? Please specify the purpose for which the property is being utilized.

Solution

1. Yes, it is true. The **reflexive property** states that any real number is equal to itself. For example, for $x=3$, we have $3=3$, which satisfies the reflexive property.
2. According to the **symmetric property**, if $a=4$, then $b=a$ implies $b=4$. Therefore, $b=4$.
3. Given $p=2$, $q=p=2$, and $r=q=2$, by the **transitive property**, we have $p=r=2$.
4. Yes, the **substitution property** allows us to replace m with 5 in any expression where m appears. For example, if $m+3$, we can substitute m with 5 to get $5+3$.

1.2. Key Questions

1. Solve the Arithmetic Problems

1) $-13 - 9 =$	2) $45 - 60 =$
3) $-9 - (-18) =$	4) $-\frac{1}{4} - \frac{3}{4} =$
5) $-124 + 56 =$	6) $32 + (-40) =$
7) $-7.55 - 3.25 =$	8) $\frac{3}{5} - \frac{4}{5} =$

2. Solve the Division Problems

1) $-230 - 65 =$	2) $68 - (-23) =$
3) $-10.75 - 5.25 =$	4) $-\frac{2}{3} - \frac{2}{3} =$
5) $-125 \div 5 =$	6) $-22 \div (-4) =$
7) $72 \div (-8) =$	8) $-\frac{1}{4} \div \left(-\frac{3}{8}\right) =$

3. Solve the following

1) $81.94 + 27.39$	2) $64.07 - 13.67$
3) $92.5 + 48.75$	4) $75.32 - 19.84$

4. Solve the following

1) 18.3×1.5	2) $358.7 \div 3.4$
3) 24.6×2.5	4) $624.8 \div 5.2$

5. Find the GCF

1) 24 and 72

2) 16 and 80

6. Find the LCM:

1) 6 and 14

2) 3 and 8

7. It takes 6 hours to travel 372 miles. How long will it take to travel 279 miles? (Travel Time Calculation)

8. Factor Out the Common Terms

1) $8x - 12y + 16$	2) $10a + 7b - 13a - 4b$
3) $-5a - 8a + 2b - b$	4) $-5a - a + 2b - 6b$

9. Test Score Percentage

1) A student gets 15 out of 25 questions correct on a test. What percentage of questions did the student get correct?
2) A student gets 18 out of 30 questions correct on a test. What percentage of questions did the student get correct?

10. Simplify the Expression

1) $3(4x - 6y) =$

2) $4(2x - 3y) =$

11. Combine Like Terms

1) $2x + 2x - x + 2y + 2y - 2z + z =$

2) $3x + 3x - 2x + 3y + 3y - 3z + 2z =$

12. Evaluate the Expression

1) $2^2 + 4(7 - 2) - 8$

2) $19 - 2(18 \div 3) + 3^2$

13. Solve the Following in Reduced Form

3) $\frac{4}{5} \div \frac{2}{9} =$

4) $\frac{7}{8} \div \frac{2}{5} =$

14. Write an Equation for the Situation

1) Brand A costs \$50.00, and brand B costs \$45.00. What is the difference in cost?

2) A theater rents their space for \$200 per night. If the company charges \$10 per ticket, what is the least number of tickets that must be sold to make a profit?

15. Calories Burned Running

Amanda is training for a marathon and wants to track the number of calories she burns while running. She notices that the number of calories burned is directly proportional to the distance she runs. After conducting some experiments, she finds that she burns 120 calories for every mile she runs. If she runs 6.5 miles, how much calories she burns?

16. Mean and Median of Grades, 70, 85, 90, 75, 95, 88

1) Find the mean

2) Find the median

1.3. Solutions

1. Solve the Arithmetic Problems

- 1) $-13 - 9 = -22$
- 2) $45 - 60 = -15$
- 3) $-9 - (-18) = 9$
- 4) $\frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$
- 5) $-124 + 56 = -68$
- 6) $32 + (-40) = -8$
- 7) $-7.55 - 3.25 = -10.8$
- 8) $\frac{3}{5} - \frac{4}{5} = -\frac{1}{5}$

2. Solve the Division Problems

- 1) $-230 \div -65 = -295$
- 2) $68 \div (-23) = 91$
- 3) $-10.75 \div -5.25 = -16$
- 4) $\frac{2}{3} \div \frac{2}{3} = \frac{4}{3} = -1\frac{1}{3}$
- 5) $-125 \div 5 = -25$
- 6) $-22 \div (-4) = 5\frac{1}{2} = 5.5$
- 7) $72 \div (-8) = -9$
- 8) $-\frac{1}{4} \div \left(-\frac{3}{8}\right) = \frac{1}{4} \times \left(\frac{8}{3}\right) = \frac{2}{3}$

3. Solve the following

- 1) $81.94 + 27.39$
 - Align the decimal points.
 - Add the numbers column by column.
 - Answer: 109.33
- 2) $64.07 - 13.67$
 - Align the decimal points.
 - Subtract the numbers column by column.
 - Answer: 50.40
- 3) $92.5 + 48.75$
 - Align the decimal points.
 - Add the numbers column by column.
 - Answer: 141.25
- 4) $75.32 - 19.84$

- Align the decimal points.
- Subtract the numbers column by column.
- Answer: 55.48

4. Solve the following

- 1) 18.3×1.5
 - Answer: 27.45
- 2) $3.4 \div 358.7$
 - Answer: 105.5
- 3) $3) 24.6 \times 2.5$
 - Answer: 61.5
- 4) $4) 5.2 \div 624.8$
 - Answer: 120.15

5. Find the GCF

- 1) 24 and 72
 - List factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
 - List factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
 - Highest common factor: 24
 - Or use below: $\text{GCF} = 2 \times 2 \times 2 \times 3 = 24$

2	24	72
2	12	36
2	6	18
3	3	9
	1	3

- 2) 16 and 80
 - List factors of 16: 1, 2, 4, 8, 16
 - List factors of 80: 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
 - Highest common factor: 16

8	16	80
2	2	10
	1	5

6. Find the LCM

- 1) 6 and 14
 - Prime factorization of 6: 2×3

- Prime factorization of 14: 2×7
- LCM: $2 \times 3 \times 7 = 42$



2) 3 and 8

- 1) Prime factorization of 3: 3
- 2) Prime factorization of 8: 2^3
- 3) LCM: $3 \times 2^3 = 24$

7. Travel Time Calculation

- Step 1: Find the speed:
 $372 \text{ miles} \div 6 \text{ hours} = 62 \text{ miles/hour}$
- Step 2: Calculate time for 279 miles:
 $279 \text{ miles} \div 62 \text{ miles/hour} = 4.5 \text{ hours}$
- Answer: 4.5 hours

8. Factor Out the Common Terms

- 1) Factor: $8x - 12y + 16$
 - Answer: $4(2x - 3y + 4)$
- 2) Factor: $10a + 7b - 13a - 4b$
 - Answer: $-3a + 3b = -3(a - b)$
- 3) Factor: $-5a - 8a + 2b - b$
 - Answer: $-13a + b$
- 4) Factor: $-5a - a + 2b - 6b$
 - Answer: $-6a - 4b = -2(3a + 2)$

9. Test Score Percentage

- 1) A student gets 15 out of 25 questions correct on a test. What percentage of questions did the student get correct?
 - Divide correct answers by total questions: $15 \div 25 = 0.6$
 - Convert to percent: $0.6 \times 100 = 60\%$
- 2) A student gets 18 out of 30 questions correct on a test. What percentage of questions did the student get correct?
 - Divide correct answers by total questions: $18 \div 30 = 0.6$
 - Convert to percent: $0.6 \times 100 = 60\%$

10. Simplify the Expression

- 1) Simplify: $3(4x - 6y)$
 - Distribute the 3: $12x - 18y$
- 2) Simplify: $4(2x - 3y)$
 - Distribute the 4: $8x - 12y$

11. Combine Like Terms

- 1) Combine: $2x + 2x - x + 2y + 2y - 2z + z$
 - Combine x terms: $3x$
 - Combine y terms: $4y$
 - Combine z terms: $-z$
 - Answer: $3x + 4y - z$
- 2) Combine: $3x + 3x - 2x + 3y + 3y - 3z + 2z$
 - Combine x terms: $4x$
 - Combine y terms: $6y$
 - Combine z terms: $-z$
 - Answer: $4x + 6y - z$

12. Evaluate the Expression

- 1) Evaluate: $2^2 + 4(7 - 2) - 8$
 - Calculate power: 4
 - Calculate inside parenthesis: 5
 - Multiply and subtract: $4 + 20 - 8 = 16$
- 2) Evaluate: $19 - 2(18 \div 3) + 3^2$
 - Divide: 6
 - Calculate power: 9
 - Multiply, subtract, and add:
 $19 - 12 + 9 = 16$

13. Solve the Following in Reduced Form

- 1) $\frac{4}{5} \div \frac{2}{9} =$
 - Multiply by reciprocal: $\frac{4}{5} \times \frac{9}{2}$
 - Simplify: $\frac{36}{10}$
 - Reduce: $\frac{18}{5}$
- 2) $\frac{7}{8} \div \frac{2}{5} =$
 - Multiply by reciprocal: $\frac{7}{8} \times \frac{5}{2}$

- Simplify: $\frac{35}{16}$

14. Write an Equation for the Situation

- 1) Steps:
 - Equation: $50.00 - 45.00 = 5.00$
 - Difference in cost: \$5.00
- 2) Steps:
 - Cost to rent: \$200
 - Ticket price: \$10
 - Inequality: $10x > 200$
 - Solve for x : $x > 20$
 - At least 21 tickets need to be sold.

15. Calories Burned Running

- Independent quantity: Miles run
- Dependent quantity: Calories burned
- Equation: Calories burned = $120 \times \text{Miles}$

- Solve: For 6.5 miles,
 $\text{Calories} = 120 \times 6.5 = 780$ calories

16. Mean and Median of Grades, 70, 85, 90, 75, 95, 88

- 1) Mean of Grades:
 - Find the mean of 70, 85, 90, 75, 95, 88 .
 - Answer: Mean =
$$\frac{70 + 85 + 90 + 75 + 95 + 88}{6} = \frac{503}{6} = 83.83$$
- 2) Median of Grades:
 - Use the same data to find the median.
 - Answer: Sort: 70, 75, 85, 88, 90, 95 .
$$\text{Median} = \frac{85 + 88}{2} = 86.5$$

Appendix. Prealgebra Foundation Concepts

1. Reading Decimals

- 0.25 is read as "zero point two five."
- 3.142 is read as "three point one four two."
- 32.094 is read as " thirty-two point zero nine four" or "thirty-two and ninety-four thousandths."

2. Comparing Decimals and Mathematical Notation

- 0.5 is greater than 0.45: $0.5 > 0.45$
- 0.7 is greater than or equal to 0.6: $0.7 \geq 0.6$
- 0.75 is less than 0.8. $0.75 < 0.8$

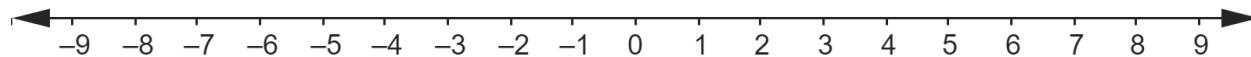
3. Converting Numbers

- Convert 50% to a decimal. Answer: 0.5
- Convert 0.25 to a percentage. Answer: 25%

4. Properties of Numbers: Commutative & Distributive

- Commutative property: $3 + 5 = 5 + 3$
- Distributive property: $a(b + c) = ab + ac$

6. The Number Line



- On a number line, -2 is to the left of 1
- On a number line, 0 is to the right of -1

5. Negative Number

- -3 is a negative number
- The opposite of 5 is -5

7. Greater Than or Less Than in Negative System

- -4 is less than -2 : $-4 < -2$
- -1 is greater than -100 : $-1 > -100$ or $-100 < -1$

8. Addition Involving Negative Numbers

- $-3 + 2 = (-1)$
- $-7 + 4 = (-3)$

9. Subtraction Involving Negative Numbers

- $-5 - (-3) = (-2)$
- $2 - (-3) = 5$

10. Multiplication and Division Involving Negative Numbers

- $-4 \times (-2) = 8$ and $-4 \div (-2) = 2$
- $-8 \div 2 = (-4)$

11. Numerator and Denominator

- In $\frac{3}{4}$, 3 is the numerator, and 4 is the denominator.
- In $\frac{5}{7}$, 5 is the numerator, and 7 is the denominator.

12. Simplifying Fractions

- "Simplifying fractions" and "reducing fractions" essentially refer to the same process, which involves making fractions easier to understand and work with by expressing them in their simplest form.
- $4/8$ simplifies to $1/2$.
- $10/20$ simplifies to $1/2$.

13. Equivalent Fractions

- $2/4$ is equivalent to $1/2$
- $3/6$ is equivalent to $1/2$

14. Comparing Fractions

- $1/2$ is greater than $1/4$: $\frac{1}{2} > \frac{1}{4}$
- $3/4$ is greater than $2/3$: $\frac{3}{4} > \frac{2}{3}$

15. Basic Reducing Fractions

- $6/8$ reduces to $3/4$
- $8/10$ reduces to $4/5$

16. Negative Signs in Fractions

- $-3/4$ means the fraction is negative
- The fraction $-5/7$ indicates a negative value

17. Unreducing Fractions

- $1/2$ can be unreduced to $2/4$
- $2/3$ can be unreduced to $4/6$

18. Converting Between Integers and Fractions

- 4 can be written as $4/1$
- 7 can be converted to $7/1$

19. Basic Multiplying Fractions

- $\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8}$
- $\frac{3}{5} \times \frac{2}{3} = \frac{3 \times 2}{5 \times 3} = \frac{6}{15}$, which simplifies to $\frac{2}{5}$

20. Reduce While You Multiply

- $\frac{2}{6} \times \frac{3}{9} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ after reduction
- $\frac{4}{8} \times \frac{6}{10} = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$ after reduction

21. Multiplying Fractions Involving Negative Numbers

- $(-1/2) * (2/3) = (-1/3)$
- $(-3/5) * (-5/7) = 3/7$

22. Multiplying a Fraction by Whole Number

- $3 \times \frac{1}{4} = \frac{3}{1} \times \frac{1}{4} = \frac{3}{4}$
- $4 \times \frac{3}{8} = \frac{4}{1} \times \frac{3}{2 \times 4} = \frac{3}{2}$

23. Dividing Fractions

- $(1/2) \div (1/4) = \frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{2}{1} = 2$
- $(3/4) \div (1/2) = 3/2$

24. Divisions Involving Fractions and Integers

- $(1/2) \div 2 = \frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
- $(3/5) \div 3 = 1/5$

25. Simplifying Compound Fractions

- $(1/2) / (3/4) = \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$
- $(4/5) / (2/3) = 6/5$

26. Adding Fractions with the Same Denominator

- $1/4 + 2/4 = \frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$
- $3/8 + 5/8 = 8/8$, which simplifies to 1

27. Subtracting Fractions

- $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$
- $\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$

28. Adding and Subtracting Fractions Involving Negative Numbers

- $\frac{-3}{5} + \frac{2}{5} = \frac{-1}{5}$
- $\frac{-4}{7} - \frac{-2}{7} = \frac{-4}{7} + \frac{2}{7} = \frac{-2}{7}$

29. Adding Subtracting Fractions and Integers

- $3 + \frac{1}{4} = 3\frac{1}{4}$
- $5 - \frac{2}{3} = 4 + \frac{3}{3} - \frac{2}{3} = 4 + \frac{1}{3} = 4\frac{1}{3}$

30. Finding Common Denominators

- For $\frac{1}{3}$ and $\frac{2}{5}$, common denominator is 15
- For $\frac{3}{4}$ and $\frac{5}{6}$, common denominator is 12

31. What is a Mixed Number?

- $3\frac{1}{2}$ is a mixed number.
- $\frac{13}{4}$ is **not** a mixed number.

32. Converting between Mixed Numbers and Fractions

- $2\frac{1}{2} = 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$
- $\frac{7}{3} = 2\frac{1}{3}$

33. Adding and Subtracting Mixed Numbers

- $3\frac{1}{2} + 2\frac{2}{3} = 6\frac{1}{6}$
- $4\frac{3}{4} - 1\frac{1}{2} = 3\frac{1}{4}$

34. Multiplying Mixed Numbers

- $2\frac{1}{4} \times 1\frac{1}{3} = \frac{9}{4} \times \frac{4}{3} = \frac{3}{1} = 3$
- $3\frac{1}{2} \times 2 = \frac{7}{2} \times 2 = 7$

35. Dividing Mixed Numbers

- $4\frac{1}{2} \div 2 = \frac{9}{2} \times \frac{1}{2} = \frac{9}{4} = 2\frac{1}{4}$
- $3\frac{3}{4} \div 1\frac{1}{2} = \frac{15}{4} \div \frac{3}{2} = \frac{15}{4} \times \frac{2}{3} = \frac{5}{2} = 2\frac{1}{2}$

36. Comparing Mixed Numbers

- $2\frac{1}{2} > 2\frac{1}{4}$
- $-2\frac{1}{2} < -2\frac{1}{4}$
- $3\frac{1}{3} < 3\frac{1}{2}$

37. Equality or Inequality of Two Fractions

- $\frac{1}{2} = \frac{2}{4}$
- $\frac{3}{4} > \frac{2}{4}$

38. Solving Proportions (Ratios)

- $\frac{2}{3} = \frac{x}{9} \Rightarrow 2 \times 9 = 3x, x = 6$
- $\frac{4}{5} = \frac{8}{x} \Rightarrow 4x = 8 \times 5, x = 10$

39. Direct (Linear) Variation in Similar Figures

- If one side of a triangle is doubled, the corresponding side of a similar triangle is also doubled.

40. Converting a Percentage to a Fraction

- $50\% = 50 \times \frac{1}{100} = \frac{1}{2}$
- $25\% = 25 \times \frac{1}{100} = \frac{1}{4}$

41. Converting a Fraction to a Percentage

- $\frac{1}{2} = \frac{1}{2} \times 100\% = 50\%$
- $\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%$

42. Converting between a Percentage and a Decimal

- $50\% = 50 \times \frac{1}{100} = \frac{5}{10} = 0.5$
- $0.25 = 25\%$

43. Word Problems Involving Percentages

- If 50% of a number is 25, the number is 50
- If you save 20% of \$200, you save \$40
- If a shirt costs \$50 and is on sale for 25% off, the sale price is \$37.50

44. Decimal Number

- 0.75
- 3.14

45. Converting Fractions to Decimals

- $\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5$
- $\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$

46. Converting Decimals to Fractions

- $0.5 = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$
- $0.25 = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$

47. Adding and Subtracting Decimal Numbers

- $0.75 + 0.25 = 1.00$
- $1.5 - 0.75 = 0.75$

48. Multiplying Decimal Numbers

- $0.5 \times 2 = 1.0$
- $0.25 \times 4 = 1.0$

49. Dividing Decimal Numbers

- $1.0 \div 2 = 0.5$
- $1.0 \div 4 = 0.25$

50. Rounding Numbers

- Round 0.75 to the nearest whole number = 1
- Round 2.34 to the nearest tenth = 2.3
- 0.746 rounded to the nearest hundredth is 0.75

51. Exponents

- $2^3 = 8$
- $3^2 = 9$

52. Prime Numbers & Composite Numbers

- 5 is a prime number
- 4 is a composite number. $4 = 2 \times 2$

53. Factor Trees

- Factor tree for $6 = 2 \times 3$
- Factor tree for $8 = 2 \times 2 \times 2$
- Factor tree for $12 = 2 \times 2 \times 3$

54. Greatest Common Factor (GCF)

- GCF of 12 and 16: $(2 \times 2) = 4$
- GCF of 18 and 24: $(2 \times 3) = 6$

2	12	16	2	18	24
2	6	8	3	9	12
3	4			3	4

55. Least Common Multiple (LCM)

- LCM of 4 and 5 = 20
- LCM of 16 and 24: $(2 \times 2 \times 2 \times 2 \times 3) = 48$
- LCM of 18 and 24: $(2 \times 3 \times 3 \times 4) = 72$

2	16	24	2	18	24
2	8	12	3	9	12
2	4	6		3	4
	2	3			

56. Purpose of Scientific Notation

- To express very large numbers, like the distance between stars.
- To express very small numbers, like the mass of a virus.
- Scientific notation is used to easily represent very large or very small numbers.

57. Reading Scientific Notation

- 3.0×10^4 is read as “three times ten to the fourth power.”
- 5.67×10^{-3} is read as “five point six seven times ten to the power of negative three.”

58. Writing Numbers in Scientific Notations

- Write 45,000 in scientific notation. 4.5×10^4
- Write 0.007 in scientific notation. 7×10^{-3}
- 5,000 can be written as 5×10^3

59. Multiplying and Dividing in Scientific Notation

- Multiply $(2 \times 10^3) \times (3 \times 10^4) = 6 \times 10^7$
- Multiply $(2 \times 10^3) \times (3 \times 10^{-2}) = 6 \times 10 = 60$
- Divide $(4 \times 10^5) \div (2 \times 10^3) = 2 \times 10^2$

60. Adding and Subtracting in Scientific Notation

- Add $(2 \times 10^3) + (3 \times 10^3) = 5 \times 10^3$
- Subtract $(5 \times 10^3) - (3 \times 10^3) = 2 \times 10^3$

61. Product, Factors, Factoring and Prime

- Factors for 12 are 1, 2, 3, 4, 6, & 12
- 2 and 3 are prime numbers
- The product of 2 and 3 is 6
- Factors of 6 are 1, 2, 3, and 6
- 6 can be factored into 2×3

62. Factor a Natural Number

- To factor 30, the prime factorization is $2 \times 3 \times 5$
- Factors of 30 are 1, 2, 3, 5, 6, 10, 15, & 30

63. Divisible by 2, 3, 4, 5, 9, or 10.

- Is 14 divisible by 2? Yes
- Is 33 divisible by 3? Yes

64. Prime Numbers

- A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself, like 7.
- Is 11 prime? Yes
- Is 21 prime? No

65. Expressions, Grouping, and Parentheses:

- Simplify $(2 + 3) \times 4 = 20$
- Simplify $6/(1 + 2) = 2$
- In the expression $2 \times (3 + 4)$, parentheses indicate that you should add 3 and 4 first.

66. Order of Operations: PEMDAS (Parentheses, Exponents, Multiplication and Division, Addition and Subtraction) dictates the order of operations.

- Calculate $2 + 3 \times 4 = 14$
- Calculate $(2 + 3) \times 4 = 20$

67. Evaluating Expressions Using Substitution:

- Evaluate $y = 2x + 3$ when $x = 5$. $y = 13$
- Evaluate $x = 3y^2 - 1$ when $y = 2$. $x = 11$
- If $x = 3$, then $2x + 4$ evaluates to $2(3) + 4 = 10$

68. Using Formulas

- Use the formula for the area of a rectangle ($A = l \times w$) to find the area when $l = 5$ and $w = 4$:
 $A = 20$
- Use the formula for the circumference of a circle ($C = 2\pi r$) to find the circumference when $r = 3$: $C = 6\pi$

69. Naming the Variables in a Word Problem

- If a problem asks for the number of apples and you let a represent this number, then a is the variable for the number of apples.
- "John has X apples, and he buys 5 more." X is a variable.
- "The total cost C is determined by multiplying the number of items N by the price per item p ." C & N are variables.

70. Translating Some Common English Phrases into Mathematics:

- "The sum of a number and 7" translates to $x + 7$.
- "Twice a number" translates to $2x$.

71. Metric System Reference

- The basic unit of length is the meter.
- The basic unit of mass is the kilogram.

72. Converting Units

- Convert 1000 meters to kilometers: 1 km
- Convert 2000 grams to kilograms: 2 kg

73. Finding Area and Perimeter

- Find the area (A) of a rectangle with length 5 cm and width 4 cm: $A = 5 \times 4 = 20 \text{ cm}^2$
- Find the perimeter (P) of a square with side length 3 cm: $P = 3 \text{ cm} \times 4 = 12 \text{ cm}$

74. The Metric System vs. The English System

- In the metric system, distance is measured in meters, whereas in the English system, it is measured in feet.
- In the metric system, weight is measured in kilograms, whereas in the English system, it is measured in pounds.

75. Measurement Conversions

- Convert 5 miles to kilometers (1 mile = 1.60934 km): Approximately 8.0467 km
- Convert 2 liters to gallons (1 gallon = 3.78541 liters): Approximately 0.528344 gallons
- Convert 10 inches to centimeters (1 inch = 2.54 cm): 25.4 cm
- Convert 3 feet to meters (1 foot = 0.3048 meters): 0.9144 meters

76. Graphing Statistics

- Create a **bar graph** to show the number of pets owned by students in a classroom.
- Create a **pie chart** to represent the percentage of each category of expenses in a monthly budget.

77. Mean and Mode: The mean is the average of a set of numbers, and the mode is the most frequently occurring number in a set.

- Find the mean of the numbers 2, 4, 6, 8, 10: Mean = 6
- Find the mode of the numbers 1, 2, 2, 3, 4: Mode = 2

78. The Counting Principle

- If there are 3 ways to choose a shirt and 2 ways to choose pants, there are $3 \times 2 = 6$ ways to choose an outfit.
- If there are 3 types of sandwiches and 2 types of drinks, how many combinations can be made? 6 combinations

79. Permutations

- "Permutations" refer to the different ways you can arrange a set of items or elements.
- The number of ways to arrange 3 books on a shelf is $3! = 3 \times 2 \times 1 = 6$ ways.
- How many ways can 4 students stand in a line? $4! = 4 \times 3 \times 2 \times 1 = 24$ ways

80. Unit Conversions

- 1 foot = 12 inches
- 1 yard = 3 feet.
- 1 mile = 5,280 feet.
- 1 pound = 16 ounces.
- 1 ton = 2,000 pounds.
- 1 gallon = 4 quarts.
- 1 quart = 2 pints = 4 cups.
- 1 cup = 8 fluid ounces.
- 1 year = 365 days.
- 1 week = 7 days.
- 1 day = 24 hours.
- 1 hour = 60 minutes.
- 1 minute = 60 seconds.

eSpyMath: Prealgebra

End of the Workbook

Congratulations!

I'm glad I could assist you in gaining math knowledge for Prealgebra. If you have any more questions or need further assistance in the future, feel free to ask.

Good luck with your studies!

About Authors

Seonwan Myung, Ph.D.

Seonwan Myung holds a Ph.D. and master's degrees in industrial engineering and has a keen interest in making math easy to learn and study. With extensive experience in math teaching and tutoring, he specializes in designing enterprise applications, particularly in software design.

Jaden Myung

Jaden Myung is a high school freshman with impressive achievements in mathematics. He completed an AP Precalculus course in 2023, excelled in Pre-Algebra and Algebra, received Academic Excellence awards in math, and won the Pythagoras Award in 2022 and 2023. These accomplishments showcase his strong mathematical skills and dedication to academic success.